Universal spectral statistics of Andreev billiards: Semiclassical approach

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The symmetry classification of complex quantum systems has recently been extended beyond the Wigner-Dyson classes. Several of the novel symmetry classes can be discussed naturally in the context of superconducting-normal hybrid systems such as Andreev billiards and graphs. In this paper, we give a semiclassical interpretation of their universal spectral form factors in the ergodic limit.

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I. INTRODUCTION

Based on early work of Wigner [1], Dyson [2] proposed a classification of complex quantum systems according to their behavior under time reversal and spin rotations. The ergodic limits of the proposed symmetry classes are described by the Gaussian orthogonal, unitary, and symplectic ensembles (GOE, GUE, and GSE) of random-matrix theory. These were initially motivated by atomic nuclei and have since been applied successfully to a large variety of systems, most notably chaotic and disordered quantum systems [3]. More recently, an additional seven symmetry classes have been identified [4], which are naturally realized in part by Dirac fermions in random gauge fields (chiral classes) [5] and in part by quasiparticles in disordered mesoscopic superconductors [6] or superconducting-normalconducting (SN) hybrid systems [7]. The common feature of the new symmetry classes is a mirror symmetry in the spectrum: if E is in the spectrum, so is $-E$. The corresponding Gaussian random-matrix ensembles differ from the Wigner-Dyson ensembles in so far as their spectral statistics, while still universal, is no longer stationary under shifts of the energy due to additional discrete symmetries.

Much insight into the range of validity of the Wigner-Dyson random-matrix ensembles has been gained from the semiclassical approach to the spectral statistics of chaotic quantum systems, based on Gutzwiller's trace formula [8]. In a seminal paper [9], Berry gave a semiclassical derivation of the spectral form factor of chaotic quantum systems for the Wigner-Dyson ensembles, partially reproducing the results of random-matrix theory and clarifying its limitations. In this paper we provide such a semiclassical interpretation, based on Andreev systems, of a generalized form factor for the Gaussian random-matrix ensembles associated with the new symmetry classes termed C and CI (the pertinent Gaussian ensembles will be referred to as C-GE and CI-GE).

There have been several attempts to apply semiclassical theory to SN hybrid systems [10–12]. Melsen *et al.* [10] pointed out that the gap induced by the proximity effect in a billiard coupled to a superconducting lead [Andreev billiard, cf. Fig. 1(b)] is sensitive to whether the classical dynamics of the (normal) billiard is integrable or chaotic. These authors showed that the proximity-induced hard gap in the chaotic case is *not* fully reproduced by semiclassical theory, the reasons for which have been discussed further in Ref. [11]. All these systems have in common that due to the presence of the superconductor the combined electron-hole dynamics is no longer chaotic even if the corresponding normal (nonsuperconducting) billiard is chaotic [13]. By contrast, we focus here on a semiclassical approach to SN hybrid systems where the combined electron-hole dynamics remains chaotic even in the presence of the superconductor. Such systems exhibit the *universal* spectral statistics of the Gaussian random-matrix ensembles for the new symmetry classes. We identify the class of periodic orbits contributing to universal features in the density of states near the Fermi energy and show that in this case, semiclassics reproduces the spectral statistics predicted by random-matrix theory.

II. UNIVERSAL SPECTRAL STATISTICS

We briefly summarize the pertinent random-matrix results for the Gaussian ensembles corresponding to the new sym-

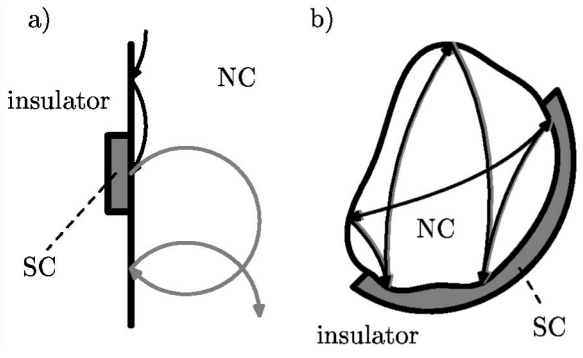

FIG. 1. (a) Andreev scattering with a perpendicular magnetic field. (b) Andreev billiard—a part of the boundary is connected to a superconductor where Andreev reflection (retroflection) takes place. An example of a self-dual orbit is shown—the SN interface is hit three times by an incoming electron and three times by an incoming hole. Each part of the trajectory is traversed twice in the same direction—once as an electron and once as a hole. The singly traversed orbit without electron-hole labels is a periodic orbit of the virtual billiard and hits the SN interface three times.

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metry classes. For the Wigner-Dyson ensembles (GUE, GOE, GSE), the average density of states is nonuniversal and random-matrix theory makes universal predictions only about spectral fluctuations in the ergodic limit such as the correlation function

$$C(\epsilon)=\langle \delta\rho(E)\,\delta\rho(E+\epsilon)\rangle \tag{1}$$

of the deviations $\delta\rho(E)$ of the density of states $\rho(E)$ from its mean value $\langle\rho(E)\rangle$. A central quantity is the spectral form factor

$$K_{\mathrm{WD}}(t)=\frac{1}{\langle\rho\rangle}\int_{-\infty}^{\infty} d\epsilon\, e^{-i\epsilon t/\hbar}C(\epsilon). \tag{2}$$

The ergodic limit of the new symmetry classes differs from the Wigner-Dyson case by the fact that even the average density of states has universal features close to the Fermi energy μ. Thus, in this case, we define a generalized spectral form factor by the Fourier transform of the expectation value of the (oscillating part of the) density of states:

$$K(t)=2\int_{-\infty}^{\infty} dE\langle\delta\rho(E)\rangle e^{-iEt/\hbar}, \tag{3}$$

where E is the energy (measured relative to μ). For the ensemble C-GE (class C is invariant under spin rotations, while time-reversal symmetry is broken), this form factor is [7]

$$K^{C}(t)=-\theta\left(1-\frac{|t|}{t_H}\right). \tag{4}$$

Here, $t_H=2\pi\hbar\rho_{\mathrm{av}}$ is the Heisenberg time defined in terms of the mean density of states ρ_{av} sufficiently far from the Fermi energy [the oscillating part of the density of states is defined as $\delta\rho(E)=\rho(E)-\rho_{\mathrm{av}}$]. Semiclassically ρ_{av} corresponds to Weyl's law. For the ensemble CI-GE (class CI differs from C by invariance under time reversal), the short-time expansion is [7]

$$K^{C\mathrm{I}}(t)=-1+\frac{|t|}{2t_H}+O(|t|^2). \tag{5}$$

Wigner-Dyson statistics can be applied even to a single chaotic system by exploiting a spectral average. By contrast, the new symmetry classes require an ensemble average since they have universal features in the vicinity of special energies (Fermi energy μ). For billiards one may average over shapes.

Before entering into the semiclassical analysis for the new symmetry classes, we briefly review the semiclassical derivation of the usual spectral form factor of the GUE. There one starts from the Gutzwiller trace formula that relates the oscillatory contribution $\delta\rho(E)$ to the density of states to a sum over periodic orbits p,

$$\delta\rho(E)=\frac{1}{\pi\hbar}\mathrm{Re}\sum_p\, t_p A_p e^{iS_p/\hbar}. \tag{6}$$

Here, S_p denotes the classical action of the orbit, A_p denotes its stability amplitude, and t_p is the primitive orbit traversal time. The explicit factor t_p arises because the traversal of the periodic orbit can start anywhere along the orbit. Inserting this expression into the definition of the spectral form factor, and employing the diagonal approximation, one finds

$$K_{\mathrm{WD,diag}}(t)=\sum_p \frac{t_p^2}{t_H}|A_p|^2\delta(t-t_p). \tag{7}$$

Finally, averaging over some time interval Δt and using the Hannay–Ozorio-de-Almeida sum rule [14] $\Sigma_p^{t_p\in[t,t+\Delta t]}|A_p|^2=\Delta t/t$ one obtains the result

$$K_{\mathrm{WD,diag}}(t)=\frac{t}{t_H} \tag{8}$$

valid for $t_0\ll t\ll t_H$, where t_0 is the period of the shortest periodic orbit. This result agrees with the short-time behavior of the spectral form factor predicted by the GUE.

III. SEMICLASSICAL APPROACH TO MAGNETIC ANDREEV BILLIARDS

We now turn to Andreev billiards—the central theme of this paper. The novel element in SN hybrid systems is Andreev reflection converting electrons into holes (and vice versa) at the interface to the superconductor [see Fig. 1(a)]. In this process, the incoming electron (hole) acquires a phase $-ie^{-i\alpha}$ $(-ie^{i\alpha})$, where α is the phase of the superconducting order parameter Δ [15]. In the absence of a magnetic field, electrons (holes) sufficiently close to the Fermi energy $(E\ll|\Delta|\ll\mu)$ are reflected as holes (electrons), which then retrace the electron (hole) trajectory backwards (retroflection). In chaotic billiards, essentially all trajectories eventually hit any given part of the boundary. Thus, if the billiard is then coupled to a superconductor any quasiparticle eventually hits the superconducting interface, leading to a periodic orbit bouncing back and forth between two points on the superconducting interface. It follows that a conventional chaotic billiard (without magnetic field) that is coupled to a superconductor has a combined electron-hole dynamics that is no longer chaotic. Instead, the resulting trajectories are all periodic, leading to nonuniversal behavior such as the proximity-induced hard gap [10,11] for time-reversal invariant systems.

One expects to recover universal spectral statistics *only* if the *combined* electron-hole dynamics is chaotic and periodic orbits are isolated as in conventional chaotic (hyperbolic) systems. In Andreev billiards this occurs naturally when time-reversal symmetry is broken by a perpendicular magnetic field (symmetry class C). In this case the retroflected hole (electron) does not retrace the trajectory of the incoming electron (hole), as electron and hole trajectories are curved in the same direction [cf. Fig. 1(a)]. This allows one to express the density of states semiclassically by a Gutzwiller-type trace formula as a sum over the isolated periodic orbits of the *combined* electron-hole dynamics of the Andreev billiard,

$$\delta\rho(E)=\frac{1}{\pi\hbar}\mathrm{Re}\sum_p t_p A_p e^{iS_p(E)/\hbar+i\chi}. \tag{9}$$

The orbit amplitudes A_p are products of electron and hole contributions,

$$A_p=A_p^{(e)}A_p^{(h)}, \tag{10}$$

while the orbit actions are sums of electron and hole actions,

$$S_p(E)=S_p^{(e)}(E)+S_p^{(h)}(E). \tag{11}$$

The factor t_p again reflects the arbitrary starting point of the orbit and χ denotes the accumulated Andreev phases.

Coherent contributions to the form the factor can be expected from periodic orbits that retrace the same trajectory in the same direction with the roles of electrons and holes interchanged. Such *self-dual* orbits are invariant under electron-hole conjugation and the dynamical contributions to their action largely cancel due to the relation $S_p^{(e)}(E)=-S_p^{(h)}(-E)$, so that $S_p(E)\simeq Et_p$. Moreover, the amplitudes of electron and hole are just complex conjugates of one another, giving $A_p=|A_p^{(e)}|^2$. The accumulated Andreev phase is $(-i)^{2s}=-1$ with s an odd integer. Keeping only the self-dual periodic orbits—the *self-dual approximation*—we find

$$\delta\rho(E)_{\mathrm{sd}}=-\frac{1}{\pi\hbar}\mathrm{Re}\sum_p t_p|A_p^{(e)}|^2 e^{iEt_p/\hbar}. \tag{12}$$

For the generalized form factor, this leads to

$$K(t)_{\mathrm{sd}}=-2\sum_{p:\mathrm{sd}} t_p|A_p^{(e)}|^2\delta(t-t_p). \tag{13}$$

This expression reveals the similarity to the diagonal approximation for the Wigner-Dyson form factors. However, only *one* factor t_p arises.

The Hannay–Ozorio-de-Almeida sum rule does not apply directly to the sum over self-dual orbits (being a sum over amplitudes of a subclass of periodic orbits). To deal with this difficulty, we introduce a *virtual billiard* with the same dynamics as the Andreev billiard except that there is *no* particle-hole conversion at the SN interface. Thus, the virtual billiard is an ordinary chaotic billiard with unusual reflection conditions at the SN interface (retroflections). Primitive periodic orbits of the virtual billiard involve either even or odd numbers of retroflections. Reintroducing electron-hole conversion, one observes that even orbits lead to non-self-dual periodic orbits in the Andreev billiard. By contrast, twofold traversals of odd orbits are periodic and self-dual in the Andreev billiard as the roles of electron and hole are interchanged in the second traversal [see Fig. 1(b)]. We can now interpret the sum over self-dual orbits in Eq. (13) as a sum over odd orbits of the virtual billiard. Since on average half of its orbits are odd, the Hannay–Ozorio-de-Almeida sum rule for the virtual billiard gives

$$\sum_{p:\mathrm{sd}}^{t_p\in[t,t+\Delta t]}|A_p^{(e)}|^2=\frac{\Delta t}{2t}. \tag{14}$$

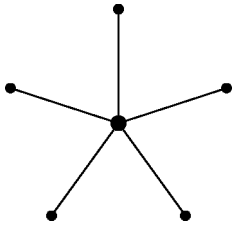

FIG. 2. Star graph with five peripheral vertices connected to superconductors.

Thus

$$K(t)_{\mathrm{sd}}=-1 \tag{15}$$

in agreement with the random-matrix result predicted by C-GE (4) for short times. The self-dual approximation is expected to hold for $t_0, t_{\mathrm{A}}\ll t\ll t_H$ where t_0 is the traversal time of the shortest periodic orbit and t_{A} the Andreev time (typical time until electron-hole conversion takes place).

IV. SPECTRAL STATISTICS FOR ANDREEV GRAPHS

The semiclassical calculation of the form factor becomes particularly transparent and explicit for quantum graphs, which were recently introduced [16] as simple quantum chaotic systems. Introducing Andreev reflection as a new ingredient, we show *semiclassically* that the form factor of the resulting *Andreev graph* takes on the universal result. A quantum graph consists of vertices connected by bonds. A particle (electron/hole) propagates freely on a bond and is scattered at a vertex according to a prescribed scattering matrix. For definiteness, we discuss star graphs with N bonds of equal length L. These have one *central* vertex and N *peripheral* vertices. Each bond connects the central vertex to one peripheral vertex (cf. Fig. 2).

Andreev (star) graphs are obtained by introducing (complete) electron-hole conversions at the peripheral vertices, while the central vertex preserves the particle type. The quantization condition is

$$\det(\mathcal{S}(k)-\mathbf{1})=0, \tag{16}$$

with the unitary $N\times N$ matrix

$$\mathcal{S}(k)=S_C\mathcal{L}D_-\mathcal{L}S_C^*\mathcal{L}D_+\mathcal{L}. \tag{17}$$

Here S_C (S_C^*) is the central scattering matrix for an electron (hole). For definiteness, we choose [17]

$$S_{C,kl}=\frac{1}{\sqrt{N}}e^{2\pi ikl/N}, \tag{18}$$

where S_C by itself does not break time-reversal symmetry. The matrix

$$\mathcal{L}=e^{ikL}\mathbf{1} \tag{19}$$

contains the phases accumulated when the quasiparticle propagates along the bonds (k is the wave number measured from the Fermi wave number). Finally,

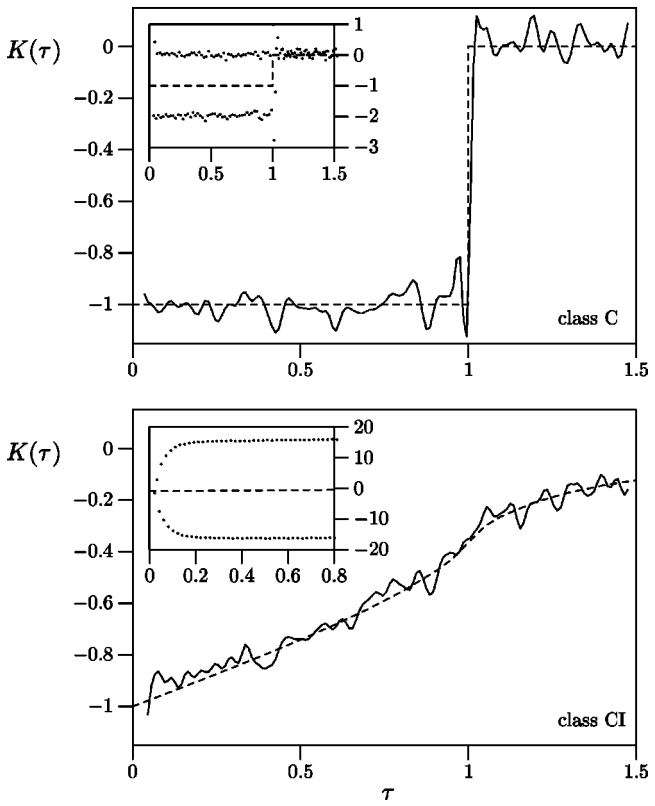

FIG. 3. Form factors for class *C* (top) and *C*I (bottom) calculated numerically for a star graph (full lines) with $N=100$ bonds (averaged over 50 000 realizations and a short-time interval $\Delta t \ll t_H$) and as obtained from the Gaussian random-matrix ensembles *C*-GE and *C*I-GE (dashed lines) in dimensionless time $\tau = t/t_H$. The insets give the coefficients K_m as a function of "time" m/N.

$$D_{\pm} = -i\,\mathrm{diag}(e^{\mp i\alpha_i}) \tag{20}$$

contains the *Andreev phases* accumulated at the vertices, where α_i denotes the order-parameter phase at peripheral vertex i. Time-reversal symmetry is obeyed if all Andreev phases are either $\alpha_i = 0$ or $\alpha_i = \pi$, but is broken otherwise. Accordingly, we build ensembles corresponding to the symmetry classes *C* (uncorrelated Andreev phases α_i with uniform distributions in the interval $[0,2\pi)$) and *C*I (uncorrelated Andreev phases taking values $\alpha_i = 0$ or $\alpha_i = \pi$ with equal probability). Numerically computed ensemble averages are in excellent agreement with random-matrix results from *C*-GE and *C*I-GE, as shown in Fig. 3.

Following previous work on quantum graphs [16], we write the density of states in k space as

$$\rho(k) = \rho_{\mathrm{av}} + \delta\rho(k), \tag{21}$$

with $\rho_{\mathrm{av}} = 2NL/\pi$ and obtain the exact trace formula

$$\delta\rho(k) = \frac{1}{\pi}\mathrm{Re}\sum_p t_p A_p e^{iS_p + i\chi} \tag{22}$$

as a semiclassical sum over periodic orbits p of the graph. Here, periodic orbits are defined as a sequence $i_1, i_2, \ldots, i_l$ of peripheral vertices, with cyclic permutations identified.

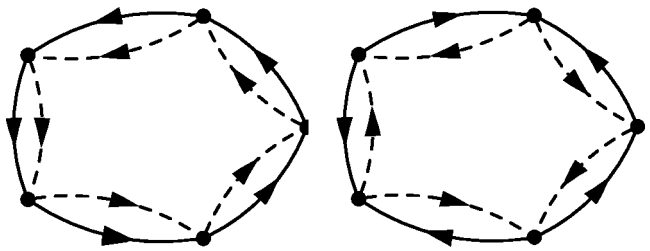

FIG. 4. Periodic orbits contributing in the self-dual approximation (at $m=5$). The vertices in the diagram correspond to peripheral vertices of the original star graph, full and dashed lines represent electron and hole propagation. In class *C*, only the left diagram contributes. In class *C*I, the right diagram gives m additional contributions as the turning point can be any of the m vertices.

Since the particle type changes at the peripheral vertices, the sequences must have even length $l=2m$. The primitive traversal "time" [18] of a periodic orbit is $t_p = 4mL/r$ (where r is the repetition number), the stability amplitude is $A_p = 1/N^m$, and the action is

$$S_p = 4mkL + \sum_{j=1}^{2m} (-1)^{j+1} 2\pi \frac{i_j i_{j+1}}{N}. \tag{23}$$

The accumulated Andreev phase is

$$\chi = -m\pi - \sum_{j=1}^{2m} (-1)^{j+1} \alpha_{i_j}. \tag{24}$$

Then, the form factor becomes

$$K(t) = 2\int_{-\infty}^{\infty} dk e^{-ikt} \langle \delta\rho(k) \rangle = \frac{t_H}{N} \sum_{m=1}^{\infty} K_m \delta\left(t - \frac{m}{N} t_H \right), \tag{25}$$

with the Heisenberg time $t_H = 4LN$. ($\langle \cdot \rangle$ denotes the average over Andreev phases.) The coefficients can be written as a sum over periodic orbits p_m with $2m$ Andreev reflections:

$$K_m = 2\sum_{p_m} \frac{m}{r} \langle A_p e^{iS_p(k=0) + i\chi} \rangle. \tag{26}$$

K_m can be viewed as a form factor in discrete time m/N.

For graphs in class *C*, only those periodic orbits survive the average over Andreev phases that visit each peripheral vertex an even number of times—half as incoming electron and half as incoming hole. In the self-dual approximation, only those orbits contribute whose total phase due to the scattering matrix of the central vertex vanishes. As the phase factors due to scattering between bonds i and j for electrons and holes are complex conjugates of one another, this requires that the periodic orbits contain equal numbers of scatterings from i to j as electron and hole. This leads to the orbits sketched in Fig. 4: An *odd* number of peripheral vertices are visited twice, once as an electron and once as a hole. First, the peripheral vertices are visited once, alternating between electrons and holes, and subsequently the vertices are visited again in the same order but with the roles of electrons

and holes interchanged. Thus, these orbits have the same structure as the self-dual orbits discussed above for the Andreev billiard. We have $A_p=1/N^m$, $S_p=4mkL$, and $\chi=m\pi$. The number of such orbits of length $2m$ is N^m/m, where the denominator m reflects the identification of cyclic permutations of peripheral vertices. With these ingredients, we find the short-time result

$$K^C_{m,\mathrm{sd}}=-1+(-1)^m \Rightarrow \overline{K^C_{\mathrm{sd}}}(t)=-1, \tag{27}$$

where $\overline{K}(t)$ is the time-averaged form factor. This reproduces the result predicted by C-GE.

For class CI, the average over the Andreev phases requires only an even number of visits to each vertex. In the self-dual approximation, this leads to additional orbits (see Fig. 4) and to the result

$$K^{C\mathrm{I}}_{m,\mathrm{sd}}=-1+(-1)^m(2m+1) \Rightarrow \overline{K^{C\mathrm{I}}_{\mathrm{sd}}}(t)=-1, \tag{28}$$

the leading order term for short times predicted by the corresponding random-matrix ensemble CI-GE.

V. ANDREEV BILLIARDS WITHOUT MAGNETIC FIELD

Finally, we come back to Andreev billiards without magnetic field (class CI). As explained above (in Sec. III), holes necessarily retrace the electron trajectory, thus leading to nonisolated periodic orbits and nonuniversal spectral statistics (hard gap). Universal spectral statistics of CI can, however, be found in such Andreev billiards with N one-channel leads. The reason for this is that Andreev billiards with N leads containing one channel each can be mapped to star graphs. The quantization condition for Andreev billiards with N leads is [12]

$$\det(\mathcal{S}(E)-\mathbf{1})=0. \tag{29}$$

Here $\mathcal{S}(E)$ is the $N\times N$ Andreev billiard scattering matrix

$$\mathcal{S}(E)=S_{NC}(E)D_-S^*_{NC}(-E)D_+, \tag{30}$$

with $S_{NC}(E)$ the scattering matrix describing the coupling of the N channels by the normal region. The matrices $D_\pm$ describing Andreev scattering in the leads are diagonal, $D_\pm=-i\,\mathrm{diag}(e^{\mp i\alpha_i})$, with a specific Andreev phase α_i for each lead. Time-reversal invariance demands $\alpha_i=0$ or $\alpha_i=\pi$. Then a detailed correspondence between billiard and star graph is obtained by substituting $\mathcal{L}S_C\mathcal{L}\rightarrow S_{NC}(E)$ and $\mathcal{L}S^*_C\mathcal{L}\rightarrow S^*_{NC}(-E)$ (with a more general central scattering matrix). Thus, the form factor of these billiards can be obtained in the self-dual approximation in complete analogy with the star graph.

VI. CONCLUSIONS

We considered the universal spectral statistics for ergodic SN hybrid systems belonging to the new symmetry classes, in the semiclassical approximation. While it was known that semiclassics has problems in some types of Andreev systems [10,11], we showed both for billiards and for quantum graphs that the universal spectral statistics of the random-matrix ensembles C-GE and CI-GE as reflected by the appropriately generalized form factor is correctly reproduced by semiclassical theory. An important condition for finding the universal spectral statistics is that the *combined* electron-hole dynamics of the Andreev system is classically chaotic. In particular, this requires that the hole does not retrace the trajectory of the incoming electron. In class C, this is naturally the case in magnetic Andreev billiards. We related the universal features in the density of states to self-dual periodic orbits, which are invariant under electron-hole exchange. Our results clarify under which conditions to expect spectral statistics described by the novel random-matrix ensembles.

The results presented can be extended to the symmetry classes D, DIII, and the chiral classes. We also note that our results for Andreev graphs remain valid for a rather large class of central scattering matrices S_C. Finally, by going beyond the self-dual approximation in Andreev graphs, it is possible to extract the orbits contributing to the form factor to linear order in t (weak localization corrections). These extensions will be discussed elsewhere [19].

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the gauge $\mathbf{A}=0$ throughout this paper. The discussion of class *C* is valid in any fixed gauge. All physical properties such as the form factor are easily shown to be gauge independent.

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